Regularity in the distribution of superclusters

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Using a measure of clustering derived from the nearest neighbour distribution and the void probability function we are able to distinguish between regular and clustered structures. With an example we show that regularity is a property of a point set, which may be invisible in the two point correlation function. Applying this measure to a supercluster catalogue [1] we conclude that there is some evidence for regular structures on large scales.

1 Introduction

Recently Einasto et al.[2] claimed that the superclusters form a quasi–regular network. A similar periodicity in the galaxy distribution with a period of $128h^{-1}$Mpc ($H = 100h$km/s/Mpc) was reported earlier from an analysis of pencil beams [3] but the statistical significance was regarded as weak. Actually this periodicity may be explained within gravitational clustering from Gaussian initial conditions [4]; see however the discussion by Szalay [5].

2 Method

To analyze the distribution of points given by the redshift coordinates of the superclusters [1] we use the spherical contact distribution $F(r)$, i.e. the *distribution function of the distance r of an arbitrary point to the nearest point of the process*. $F(r)$ is equal to the expected fraction of volume occupied by points which are not farther than r from the nearest point of the process. Therefore, $1 - F(r)$ is equal to the void probability function $P_0(r)$. As another tool we use the nearest neighbour distribution $G(r)$ which is defined as the *distribution function of distances r of a point of the process to the nearest other point of the process*. For a homogeneous Poisson process the probability to find a point only depends on the mean number density $\overline{\rho}$, leading to the well–known result

$$F(r) = 1 - \exp\left(-\overline{\rho}\frac{4\pi}{3}r^3\right) = G(r). \qquad (1)$$

To characterize the clustering of a point process, van Lieshout and Baddeley [7]

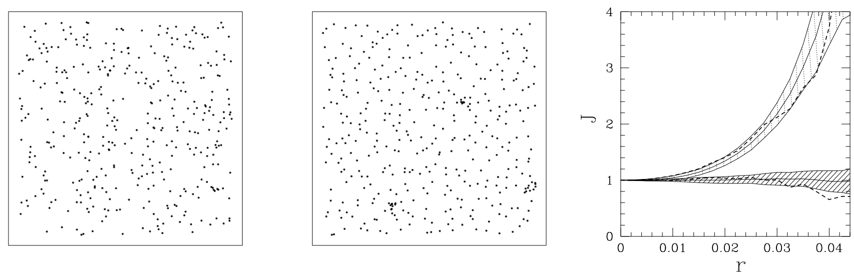

Figure 1: On the left a realization of a Poisson process and in the middle a realization of the regular example[6] is displayed. The plot on the right side shows the $J(r)$ for the Poisson process (dark dashed) and for the regular example (light dotted). The areas correspond to the 1σ error estimated from fifty realizations. The dashed lines are the $J(r)$ for the particular realizations shown in the left and middle panels, r is in units of the side length of the square.

proposed to use the ratio

$$J(r) = \frac{1 - G(r)}{1 - F(r)}. \tag{2}$$

Inherited from $F(r)$ and $G(r)$, $J(r)$ depends on correlations of arbitrary order. For a homogeneous Poisson process we get $J(r) = 1$.

A process with enhanced clumping implies $J(r) \leq 1$, whereas regular structures are indicated by $J(r) \geq 1$. Regular structures are seen for instance in a periodic, or a crystal–like arrangement of points. In a statistical sense, and opposed to clustering, regular ("ordered") structures are also seen in liquids. Qualitatively one may explain the behavior of $J(r)$ in the following way:

• In a clustered distribution of points $G(r)$ increases more rapidly with growing radius than for a Poisson process, since the nearest neighbour is typically in the close surrounding. $F(r)$ increases more slowly, since an arbitrary point is typically in between the clusters. These two effects give rise to a $J(r) \leq 1$.

• In the contrary, in a regular process, $G(r)$ is lowered with respect to a Poisson process since the nearest neighbour is typically at a finite, in the case of a crystal, characteristic distance. $F(r)$ is increasing stronger, since the typical distance from a random point to a point on a regular structure is smaller. These two effects cause a $J(r) \geq 1$.

• $J(r) = 1$ indicates the borderline between clustered and regular structures.

We illustrate these properties with two different point processes: the homogeneous Poisson process and the process constructed by Baddeley and Silverman[6]. Both processes have the same two–point characteristics, i.e. $\xi = 0$, but the example of Baddeley and Silverman is regular by construction. In Fig. 1

we display realizations of both processes with 388 points in a square. By visual inspection and with $J(r) \geq 1$, the process given by Baddeley and Silverman shows regular structures, moreover $J(r)$ clearly distinguishes between these two random processes which both have $\xi(r) = 0$. Obviously the differences in $J(r)$ result from high–order correlations only.

3 Supercluster data

We investigate the supercluster sample [1] within galactic latitude $|b| > 20°$ and a maximum radial distance of $330h^{-1}$Mpc, and perform our analysis separately for the northern and southern parts. To estimate the $F(r)$ and $G(r)$ from the data we use the reduced sample estimators [8] thereby taking care of edge effects. As seen in Fig. 2, $J(r)$ for the superclusters is clearly above one, indicating regular structures. With a nonparametric Monte Carlo test we show [8] that these results are incompatible with a random configuration of points at confidence level of 95%. However, the superclusters were identified with a friend–of–friend procedure [1] from the Abell/ACO cluster sample. Applying a friend–of–friend procedure to a random set of points we recognize with the $J(r)$ that the regularity seen in the distribution of superclusters in the northern part (galactic coordinates) may be a spurious result of the friend–of–friend algorithm. Still, the regularity seen with the $J(r) \geq 1$ in the southern part, cannot be explained with random points and a friend–of–friend procedure afterwards. Future investigations will clarify the level of significance of this regularity [8].

The difference between the northern and southern parts are not only due to the different selection criteria of the Abell and ACO parts, since large fluctuations (i.e. cosmic variance) are also seen in the galaxy distribution up to scales of $200h^{-1}$Mpc [9].

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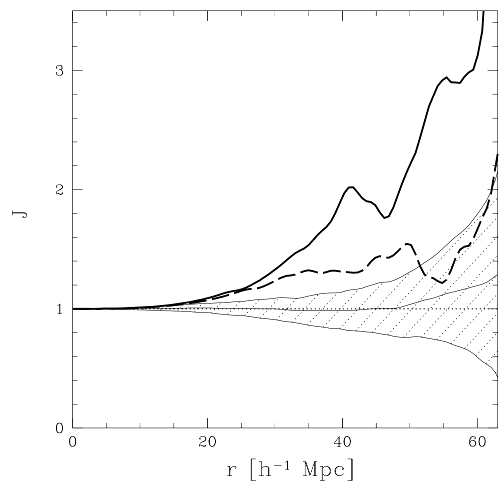

Figure 2: $J(r)$ for the supercluster catalogue (southern part: solid line, northern part: dashed line), and the 1–σ area of $J(r)$ for a Poisson Process with the same number density determined from 100 realizations.

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